Lesson 26. Applications of Integration: Probability

1   Definitions

- A **random variable** is a variable that takes on its values by chance

  - One perspective: a random variable represents unknown future results
  - e.g. the time it takes to get a pizza delivered

- Let X and Y be a pair of random variables

  - e.g. the height and weight of an adult chosen at random

- The **joint density function** of X and Y is a function f of two variables that defines the probability that (X, Y) is in some region D as follows:

- For example, when D is a rectangle $\{(X, Y) : a \leq X \leq b, c \leq Y \leq d\}$, then

- Probabilities aren't negative, so the joint density function must satisfy

- Probabilities are measured on a scale from 0 to 1, so

- The **expected value** of X is

- The **expected value** of Y is

2 Examples

Example 1. The joint density function for a pair of random variables X and Y

$$f(x, y) = \begin{cases} Cx(1+y) & \text{if } 0 \le x \le 1, 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of the constant C.
b. Find $P(X \le 1, Y \le 1)$.
c. Find $P(X + Y \le 1)$.
d. Find the expected value of Y.